VCG-Based Incentive-Compatible LQ Control for Truthful and Efficient Vehicle Platooning ⋆

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Abstract: Vehicle platooning enhances fuel efficiency, traffic flow, and safety by maintaining a common velocity and minimizing acceleration fluctuations. However, strategic misreporting in cooperative control can lead to inefficiencies. To address this, the present work proposes a pricing-based Vickrey–Clarke–Groves (VCG) mechanism that ensures truthful cost reporting, aligning individual incentives with collective goals. Applied to platooning, the approach optimizes velocity and spacing while minimizing energy use and discomfort. A supervisor solves a linear quadratic (LQ) control problem based on declared costs, with a pricing mechanism penalizing misreporting. This method is particularly relevant for freight convoys, ride-sharing fleets, and autonomous vehicle formations, where operators or autonomous agents may have competing priorities. By enforcing truthful participation, the framework enhances sustainability, efficiency, and stability in cooperative driving, benefiting commercial and public transportation networks.

Keywords: Vehicle platooning, Vickrey–Clarke–Groves (VCG) mechanism, Truthful cost reporting, Cyber-Physical Systems

1. INTRODUCTION

Recent advancements in connected and automated vehicle (CAV) technologies have paved the way for cooperative platooning—a promising approach to enhance fuel efficiency, reduce driving effort, and improve traffic stability Zhang and Li (2021). However, coordinating platoon formation presents significant challenges, particularly in selecting a platoon leader who bears a disproportionate share of the operational cost, and in designing mechanisms that are both efficient and fair Larsson and Sennton (2015). Auction-based mechanisms, particularly the Vickrey-Clarke-Groves (VCG) mechanism, have emerged as a powerful tool for such coordination due to their incentive-compatibility properties Vickrey (1961); Clarke (1971); Groves (1973). Application of the technique tO LQ control for discrete-time systems with a common setpoint of interest was developed in Angeli et al. (2025). Sun and Yin (2021) proposes a distributed auction mechanism for leader selection in single-brand platooning, showing that, while full economic properties can't all be met, an ϵ-approximate linear-transfer approach achieves near-optimality—though its focus on homogeneous fleets limits use in more diverse scenarios, Chen and Wang (2020). Complementing this, a simulation-based analysis of vehicle routing with time windows using a one-sided VCG mechanism Elbert and Roeper (2023) shows that auction-based approaches can be effectively applied to dynamic routing problems. Originally developed for public goods allocation Clarke (1971), the VCG mechanism promotes truth-telling and maximizes social welfare; applied to time window reallocation, it significantly improves throughput and routing efficiency, though its high computational complexity limits real-time use in large networks Wang and Liu (2022). Recent work by Nisan et al. (2007) further highlights the trade-offs between computational tractability and incentive alignment in VCG-based systems.

More recently, research on maximizing truck platooning participation with preferences Barua et al. (2023) has extended the discussion by incorporating heterogeneous driver and operator preferences into mechanism design. While the VCG mechanism is theoretically robust, its practical limitations in budget balance and susceptibility to collusion Ausubel and Milgrom (2006) have prompted adaptations such as the Parkes and Ungar (2001) iterative VCG auctions. By explicitly modeling preferences, the proposed approach broadens the applicability of auction mechanisms to mixed fleets and diverse operational scenarios. However, these extensions introduce additional complexity in preference aggregation, often requiring approximate equilibrium solutions Li and Zhang (2021).

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Further studies have explored game-theoretic adaptations of VCG for multi-agent systems Shoham and Leyton-Brown (2008), reinforcement learning for adaptive platoon control Guo and Peng (2022), and blockchain-based trust mechanisms for decentralized VCG implementations Kumar and Singh (2023). These works highlight the growing need for scalable and secure solutions in large-scale vehicle networks. However, gaps remain in integrating VCG-based economic mechanisms with real-time control strategies while ensuring fairness and computational tractability Bhoopalam and Agatz (2021).

Motivated by these contributions, this paper advances the literature on cooperative vehicle platooning by introducing a pricing-based VCG mechanism that addresses critical gaps in incentive alignment, heterogeneity, and real-world applicability. While prior work Sun and Yin (2021) establishes a foundational auction mechanism for homogeneous platoons, the proposed approach extends VCG principles to heterogeneous fleets, ensuring truthful cost reporting without sacrificing efficiency—overcoming the trade-offs identified in their impossibility theorem. Unlike the one-sided VCG mechanism applied to dynamic routing Elbert and Roeper (2023), which faces computational bottlenecks, the proposed method integrates a linear-quadratic (LQ) control framework to optimize velocity and spacing in real time, balancing energy efficiency and comfort while maintaining scalability Zhang and Li (2021). Furthermore, the solution explicitly enforces incentive compatibility through a rigorous VCG pricing mechanism, mitigating strategic misreporting in mixed fleets (e.g., freight convoys and ride-sharing networks) Bhoopalam and Agatz (2021). By unifying mechanism design with optimal control, this work not only complements simulation-based analysis but also enhances sustainability, stability, and fairness in cyber-physical vehicle coordination—filling a key void between theoretical auctions and real-world platoon operations.

Notice that this paper addresses strategic misreporting, a distinct, incentive-driven threat, which differs fundamentally from conventional cyber-attacks (e.g. Sybil, spoofing, denial-of-service-DoS, and man-in-the-middle-MITM) Braiteh et al. (2024). While such malicious attacks exploit identity forgery, data tampering, or communication disruption in vehicular networks, the VCG-based mechanism instead tackles truthful cost reporting by rational agents.

The paper is organized as follows: Section 2 introduces the problem framework; Section 3 formulates the LQ cost; Section 4 details the supervisor algorithm; Section 5 presents simulations; and Section 6 concludes with findings and future work.

2. PROBLEM FRAMEWORK

Consider a vehicle platooning system Σ composed of n vehicles, where each vehicle i aims to maintain a common optimal velocity v^* and a reference trajectory v^*t. Let y_i denote the position of vehicle i and $\dot{y}_i$ its velocity. The state of vehicle i is defined as $\mathbf{x}_i = [y_i, \dot{y}_i]^T$, and the collective state of the platoon as $\mathbf{x} = [\mathbf{x}_1^T, \mathbf{x}_2^T, \ldots, \mathbf{x}_n^T]^T$. The dynamics of each vehicle are described by:

$$m_i \ddot{y}_i = u_i - k_i \dot{y}_i, \qquad (1)$$

where y_i is the position of vehicle i, u_i is the control input, m_i is the vehicle mass, and k_i is a damping coefficient. A centralized supervisor S manages the control inputs u_i to ensure smooth operation. A set of n vehicles (agents) of type (1), $i = 1, \ldots, n$, participate in the system and experience costs related to velocity deviations, control effort, and relative spacing. In a platoon, vehicles benefit from aerodynamic drafting, which lowers drag and fuel consumption. When a vehicle accelerates more slowly, it increases the gap ahead, weakening the drafting effect and increasing air resistance and fuel use. Other vehicles must adjust their speeds to compensate, wasting energy through unnecessary changes. This disrupts the aerodynamic flow for the entire platoon, raising drag and overall fuel use. To restore optimal speed, the slower vehicle must accelerate inefficiently, consuming more fuel. Therefore, for each vehicle V_i, a cost function J_i is considered.

Definition 1. The function $J_i(x, u)$ represents the expense function of vehicle V_i, accounting for velocity tracking, acceleration control, and relative spacing. The total community cost is $J_0 = \sum_{i=1}^{n} J_i(x, u)$.

Assumption 2. The supervisor S does not have direct knowledge of each vehicle's exact expense function J_i and relies on self-reported values.

Prior to the supervisor S taking control of the platooning system Σ, each vehicle must submit its cost function. Each vehicle is given only a single opportunity to privately convey its cost function to S.

Definition 3. The declared (or nominal) expense functions reported by the n vehicles are denoted as $\tilde{J}_i$, $i = 1, \ldots, n$.

In platooning systems, the declared cost functions $\tilde{J}_i$ may differ from the true costs J_i. This can result from limited understanding of how expenses depend on the system state Σ, or from deliberate misreporting to influence the supervisor S for individual gain at the expense of others.

This paper explores such strategic behavior and proposes a corrective fee policy that discourages false declarations by aligning individual incentives with truthful reporting. The following sections outline the components needed to design this policy. The operational horizon $H \subset \mathbb{R}$ defines the time span over which platooning is studied and costs are incurred. The declared operation cost of vehicle V_i is defined as $\tilde{C}_i(x(\cdot), u(\cdot)) := \int_H \tilde{J}_i(x(t), u(t))\, dt$, while the overall declared operation cost of the platoon is: $\tilde{C}_0 = \sum_{i=1}^{n} \tilde{C}_i$. The corresponding actual cost of vehicle i and platoon are denoted by C_i and C_0, respectively.

Definition 4. The actual total cost experienced by vehicle V_i is $\mathcal{C}_i = C_i + p_i$, where p_i is a fee or payment imposed by the supervisor S.

3. QUADRATIC COST AND LQ FORMULATION

To simplify the equations, define the transformed variables $\tilde{y}_i = y_i - v^*t - d_i, \quad \dot{\tilde{y}}_i = \dot{y}_i - v^*, \quad \tilde{u}_i = u_i - k_i v^*$. The system dynamics transform to:

$$m_i \ddot{\tilde{y}}_i = \tilde{u}_i - k_i \dot{\tilde{y}}_i. \qquad (2)$$

In this context, the operational horizon of system Σ, as described by equation (1), is given by $H = [0, +\infty)$, meaning that its operation extends over a sufficiently long period to be effectively regarded as infinite. Furthermore, all vehicles in the platoon share the common objective of guiding Σ toward a designated final equilibrium state $\tilde{x}_{eq}$, which, without loss of generality, is set to $\tilde{x}_{eq} = 0$. Additionally, the actual cost functions are assumed to follow a quadratic form. The cost function associated with each vehicle is:

$$J_i(x, u_i) = q_i^p(y_i - v^*t - d_i)^2 + q_i^v(\dot{y}_i - v^*)^2 + r_i(u_i - k_i v^*)^2 + \gamma_i(y_i - y_{i+1} - d_i + d_{i+1})^2 + \gamma_i(y_i - y_{i-1} - d_i + d_{i-1})^2 + \beta_i(\dot{y}_i - \dot{y}_{i+1})^2 + \beta_i(\dot{y}_i - \dot{y}_{i-1})^2$$

The cost function $J_i(x, u_i)$ for vehicle i combines four critical objectives: the term $q_i^p(y_i - v^*t - d_i)^2$ penalizes deviations from the desired absolute position, where $v^*t + d_i$ is the nominal trajectory based on the target velocity v^* and the preferred position offset d_i for vehicle i; velocity tracking through $q_i^v(\dot{y}_i - v^*)^2$ to maintain target speed v^*, control effort minimization via $r_i(u_i - k_i v^*)^2$ to reduce energy consumption, safe following inter-distance enforcement with $\gamma_i(y_i - y_{i+1} - d_i + d_{i+1})^2$ for vehicle $i+1$, and proper leading spacing through $\gamma_i(y_i - y_{i-1} - d_i + d_{i-1})^2$ for vehicle $i-1$; finally the terms $\beta_i(\dot{y}_i - \dot{y}_{i+1})^2$ and $\beta_i(\dot{y}_i - \dot{y}_{i-1})^2$ promote synchronization of velocity with neighboring vehicles, enhancing string stability and reducing traffic disturbances. For boundary vehicles, it is possible to set $\gamma_i = 0$ and $\beta_i = 0$ in the non-existent coupling terms. Specifically, for the leader ($i = 1$), the term containing y_0 and $\dot{y}_0$ are omitted while for the last vehicle ($i = n$), the term containing y_{n+1} $\dot{y}_{n+1}$ are omitted.

The cost function in terms of the transformed variables is: $J_i(\tilde{x}, \tilde{u}) = q_i^p\tilde{y}_i^2 + q_i^v\dot{\tilde{y}}_i^2 + r_i\tilde{u}_i^2 + \gamma_i(\tilde{y}_i - \tilde{y}_{i+1})^2 + \gamma_i(\tilde{y}_i - \tilde{y}_{i-1})^2 + \beta_i(\dot{\tilde{y}}_i - \dot{\tilde{y}}_{i+1})^2 + \beta_i(\dot{\tilde{y}}_i - \dot{\tilde{y}}_{i-1})^2$.

Therefore, the actual expense functions are assumed to be quadratic and take the form: $J_i(\tilde{x}, \tilde{u}) = \begin{bmatrix}\tilde{x}^T & \tilde{u}^T\end{bmatrix} M_i \begin{bmatrix}\tilde{x} \\ \tilde{u}\end{bmatrix}$ for each vehicle $i = 1, \ldots, n$, where $M_i = \begin{bmatrix} Q_i & N_i \\ N_i^T & R_i \end{bmatrix}$. Although the cost function J_i fundamentally depends only on the local control input $\tilde{u}_i$, it is formally expressed as a function of the full control vector $\tilde{\mathbf{u}} = [\tilde{u}_1, \ldots, \tilde{u}_n]^T$ to maintain notational consistency with standard optimal control formulations. The matrix M_i is referred to as the actual expense matrix of vehicle V_i.

More flexible formulations can be considered in accordance with standard Linear-Quadratic (LQ) control theory. Notably, the expense functions are time-invariant, and the system state $\tilde{x}$ evolves based on the control input $\tilde{u}$ according to equation (2).

Before the supervisor begins the operations, each player V_i submits its declared expense function by privately communicating to S a matrix $\tilde{M}_i$, which is referred to as the declared expense matrix. It is important to note that $\tilde{M}_i$ may not necessarily be identical to the actual matrix M_i, which corresponds to the true expense function of player V_i. Specifically, the declared expense function can be written as $\tilde{J}_i(\tilde{x}, \tilde{u}) = \begin{bmatrix}\tilde{x}^T & \tilde{u}^T\end{bmatrix} \begin{bmatrix} \tilde{Q}_i & \tilde{N}_i \\ \tilde{N}_i^T & \tilde{R}_i \end{bmatrix} \begin{bmatrix}\tilde{x} \\ \tilde{u}\end{bmatrix}$.

Based on the information provided by all players, the supervisor computes the nominal overall expense function, which, in line with the previous notation, can be expressed as $\tilde{J}_0(x, u) = \begin{bmatrix}\tilde{x}^T & \tilde{u}^T\end{bmatrix} \tilde{M}_0 \begin{bmatrix}\tilde{x} \\ \tilde{u}\end{bmatrix}$ where $\tilde{M}_0 = \begin{bmatrix} \tilde{Q}_0 & \tilde{N}_0 \\ \tilde{N}_0^T & \tilde{R}_0 \end{bmatrix} = \sum_{i=1}^n \tilde{M}_i$. The matrix $\tilde{M}_0$ is the declared overall expense matrix. The role of the supervisor S is to compute the control input $\tilde{u}$ that minimizes the declared overall operation cost, given by $\tilde{C}_0(\tilde{x}(\cdot), \tilde{u}(\cdot)) = \sum_{i=1}^n \tilde{C}_i(\tilde{x}(\cdot), \tilde{u}(\cdot))$ which can be expressed as:

$$\tilde{C}_0(\tilde{x}(\cdot), \tilde{u}(\cdot)) = \int_0^\infty \begin{bmatrix}\tilde{x}(t)^T & \tilde{u}(t)^T\end{bmatrix} \tilde{M}_0 \begin{bmatrix}\tilde{x}(t) \\ \tilde{u}(t)\end{bmatrix} dt \qquad (3)$$

under the assumption that this value is finite.

The dependence of the expenses and integrated costs on M_i can be made explicit by defining, for any symmetric matrix M of compatible dimension $J(M, \tilde{x}, \tilde{u}) := \begin{bmatrix}\tilde{x}^T & \tilde{u}^T\end{bmatrix} M \begin{bmatrix}\tilde{x} \\ \tilde{u}\end{bmatrix}$. According to this notation, it holds that $J_i(\tilde{x}, \tilde{u}) = J(M_i, \tilde{x}, \tilde{u})$. Similarly: $C(M, \tilde{x}(\cdot), \tilde{u}(\cdot)) := \int_0^\infty J(M, \tilde{x}(t), \tilde{u}(t))\, dt$.

Thus, the related operation cost over the infinite horizon of operations H can be computed as:

$C_i = C(M_i, \tilde{x}(\cdot), \tilde{u}(\cdot)) = \int_0^\infty J(M_i, \tilde{x}(t), \tilde{u}(t))\, dt$.

Exploiting the above notation, the declared and actual costs for vehicle V_i are given by $C(\tilde{M}_i, \tilde{x}^*, \tilde{u}^*)$ and $C(M_i, \tilde{x}^*, \tilde{u}^*)$, respectively. Similarly, the declared and actual total platoon costs are $C(\tilde{M}_0, \tilde{x}^*, \tilde{u}^*)$ and $C(M_0, \tilde{x}^*, \tilde{u}^*)$.

It is well established that the supervisor S must formulate and solve a linear-quadratic optimal control problem corresponding to system (2) while considering the cost function given in equation (3). This requires determining the solution of a Continuous Algebraic Riccati Equation (CARE) with respect to the unknown matrix $\tilde{P}_0$:

$A^T\tilde{P}_0 + \tilde{P}_0 A + \tilde{Q}_0 - (\tilde{P}_0 B + \tilde{N}_0)(\tilde{R}_0)^{-1}(B^T\tilde{P}_0 + \tilde{N}_0^T) = 0$,

where A and B are the dynamical and input matrix of the overall platoon system Σ and (A, B) is assumed reachable. Solving the Algebraic Riccati Equation (ARE), the control with related optimal gain is obtained:

$$\tilde{u}^*(t) = K\tilde{x}(t), \quad K = -\tilde{R}_0^{-1}(\tilde{N}_0^T + B^T\tilde{P}_0). \qquad (4)$$

Notice that in the considered case, $\tilde{N}_0 = 0$.

4. TRUTHFUL PLATOON CONTROL VIA SUPERVISOR ENFORCEMENT

The supervisor S regulates the control input $\tilde{u}$ based on the expense functions $\tilde{C}_i$ reported by the vehicles within the platoon. Misreporting these values can influence the system's control actions, potentially allowing an individual vehicle to manipulate the platoon's behavior to its advantage. For example, by exaggerating its cost as $\tilde{M}_i = \alpha M_i$ with $\alpha > 1$, the control gain K may favor that vehicle. Since S does not have direct access to the true expense functions, the only way to ensure accurate reporting is to implement a fee mechanism that aligns each vehicle's individual incentives with the collective efficiency of the platoon. The mechanism operates in two steps:

Step 1: The supervisor S calculates the optimal control input $\tilde{u}^*(t)$ by solving an LQ control problem based on the declared cost matrices, aggregated into $\tilde{M}_0$.

Step 2: Once the corresponding state and control solutions $(\tilde{x}^*, \tilde{u}^*)$ are obtained, S implements a fee policy, charging each vehicle V_i a payment $p_i(\tilde{x}^*, \tilde{u}^*)$ to ensure that its total cost $\mathcal{C}_i = C(M_i, \tilde{x}^*, \tilde{u}^*) + p_i(\tilde{x}^*, \tilde{u}^*)$ presents the minimum for truthful reporting, thus promoting such behavior. Since $\tilde{u}^*$ and $\tilde{x}^*$ depend on $\tilde{M}_0$, they are also influenced by each vehicle's declared expense function.

Since S cannot directly verify the accuracy of the declared cost functions, it evaluates payments by assessing the effect of each vehicle's report on the total platoon cost. The objective is to discourage false declarations while ensuring fuel efficiency and stability in the platoon.

4.1 Infinite Horizon Setting

Let V_i be a vehicle with actual expense matrix M_i and declared expense matrix $\tilde{M}_i$. The sum of the declared matrices of all other vehicles is $\tilde{M}_{-i} = \sum_{j \neq i} \tilde{M}_j$. Thus, the total declared expense matrix is $\tilde{M}_0 = \tilde{M}_i + \tilde{M}_{-i}$. If S selects $\tilde{u}^*$ as the socially optimal control input, the total declared cost of all vehicles except V_i is $C(\tilde{M}_{-i}, \tilde{x}^\star, \tilde{u}^\star) = \int_0^\infty \left[\tilde{x}^\star(t)^T \tilde{u}^\star(t)^T\right] \tilde{M}_{-i} \begin{bmatrix}\tilde{x}^\star(t)\\ \tilde{u}^\star(t)\end{bmatrix} dt = x^\star(0)^T \hat{P}_{-i} x^\star(0)$, where $\hat{P}_{-i}$ satisfies the Lyapunov equation:

$$\begin{aligned}(A+BK)^T\hat{P}_{-i} + \hat{P}_{-i}(A+BK) \\ = -\tilde{Q}_{-i} - K^T\tilde{N}_{-i}^T - \tilde{N}_{-i}K - K^T\tilde{R}_{-i}K\end{aligned} \tag{5}$$

and K is chosen according to (4). The cost allocations and payment mechanisms are modeled as outcomes in a non-cooperative game framework. In this strategic setting, each player V_i selects its action by declaring a cost matrix $\tilde{M}_i$, while the collective declaration of all other players forms $\tilde{M}_{-i} = \sum_{j \neq i} \tilde{M}_j$. The resulting optimal system behavior, characterized by the state trajectory $\tilde{x}^\star$ and control input $\tilde{u}^\star$, is fundamentally determined by the composite cost structure $\tilde{M}_0 = \tilde{M}_i + \tilde{M}_{-i}$ through the optimal feedback control law $\tilde{u}^\star = K\tilde{x}^\star$. This establishes a well-defined mapping $\Gamma : (\tilde{M}_1, ..., \tilde{M}_n) \mapsto (\tilde{x}^\star_{\tilde{M}_0}, \tilde{u}^\star_{\tilde{M}_0})$ from the space of player declarations to the resulting system state-input trajectories under optimal control. To assess the impact of reporting strategies, it is possible to define each vehicle's cost as:

$$\mathcal{C}_i(\tilde{M}_i, \tilde{M}_{-i}) = C(M_i, \tilde{x}^*_{\tilde{M}_0}, \tilde{u}^*_{\tilde{M}_0}) + p_i(\tilde{x}^*_{\tilde{M}_0}, \tilde{u}^*_{\tilde{M}_0}).$$

where the cost functional is given by $C(M, x, u) = \int_0^\infty \left[x(t)^T \;\; u(t)^T\right] M \begin{bmatrix}x(t)\\ u(t)\end{bmatrix} dt$. Notice that $C(M_i, \tilde{x}^*_{\tilde{M}_0}, \tilde{u}^*_{\tilde{M}_0})$ entails the true matrix M_i and not the declaration of agent i, $\tilde{M}_i$.

The VCG payment rule is defined for any closed-loop solution $(x(t), u(t))$ as:

$$p_i(x, u) = C(\tilde{M}_{-i}, x, u) - C(\tilde{M}_{-i}, x^\star_{\tilde{M}_{-i}}, u^\star_{\tilde{M}_{-i}}) \tag{6}$$

where $(x^\star_{\tilde{M}_{-i}}(t), u^\star_{\tilde{M}_{-i}}(t))$ are the optimal trajectories for the closed-loop system with respect to the cost $\tilde{M}_{-i}$ which does not include the declarations of agent V_i. It is important to remark that the term $C(\tilde{M}_{-i}, x^\star_{\tilde{M}_{-i}}, u^\star_{\tilde{M}_{-i}})$ is not influenced by $\tilde{M}_i$. For the particular solution $(x^\star_{\tilde{M}_0}(t), u^\star_{\tilde{M}_0}(t))$ generated by the full declared cost $\tilde{M}_0$, the payment simplifies to:

$$p_i = x^\star(0)^T(\hat{P}_{-i} - \tilde{P}_{-i})x^\star(0) \tag{7}$$

where $\hat{P}_{-i}$ solves the continuous-time Lyapunov equation (5), while $\tilde{P}_{-i}$ solves the CARE for cost $\tilde{M}_{-i}$:

$$A^T\tilde{P}_{-i} + \tilde{P}_{-i}A - (\tilde{P}_{-i}B + \tilde{N}_{-i})\tilde{R}_{-i}^{-1}(B^T\tilde{P}_{-i} + \tilde{N}_{-i}^T) = \tilde{Q}_{-i}$$

Below, the fundamental properties of the VCG payment rule are outlined. Proofs are omitted for the sake of space.

Proposition 1. For any admissible solution $(x(t), u(t))$ of the platoon Σ and any vehicle V_i, the VCG payment satisfies: $p_i(x, u) \geq 0$

Moreover the VCG mechanism incentivizes truthful reporting of private costs.

Theorem 2. Consider the LQR platoon system (1) managed by a supervisor S implementing:

(1) The optimal feedback policy $u^*(t) = Kx^*(t)$ derived from the declared social cost matrix $\tilde{M}_0 = \sum_{i=1}^n \tilde{M}_i$
(2) VCG payments according to (6).

Then, for each vehicle V_i, truthful declaration $\tilde{M}_i = M_i$ constitutes a dominant strategy, satisfying:

$$\mathcal{C}_i(M_i, \tilde{M}_{-i}) \leq \mathcal{C}_i(\tilde{M}_i, \tilde{M}_{-i}) \quad \forall \tilde{M}_i, \tilde{M}_{-i} \geq 0. \tag{8}$$

Proposition 3. For any vehicle V_i, if $\tilde{M}_i = 0$, then $p_i = 0$.

This mechanism ensures that honest declarations minimize each vehicle's total cost, aligning individual incentives with the overall efficiency of the platoon while preserving stability and reducing unnecessary acceleration fluctuations.

5. SIMULATION VALIDATION

In order to validate the proposed VCG mechanism in the context of vehicle platooning, numerical simulations were conducted using MATLAB R2024b with the Control System Toolbox. These simulations were designed to demonstrate the mechanism's effectiveness in promoting truthful reporting and optimizing overall platoon performance. To guarantee well-posedness and improve the numerical feasibility of solving the associated LQ problems, a slightly larger positive definite matrix, ϵI, $\epsilon = 10^{-4}$ was added by the mechanism to all cost matrices. A platoon of five vehicles was considered, each with distinct parameters for mass, damping coefficient, and cost function weights. The vehicle dynamics were modeled using the equations provided in Section 2. The platoon simulation parameters are summarized in Table 1.

The simulation comprised two distinct phases. In Phase I, the platoon decelerates from its cruise speed v_0 to a lower desired velocity v_I^* due to a traffic congestion. The final desired spacing $d_{ij} = d_i - d_j$ between vehicles i and $j = i + 1$, $i = 1, \ldots, 4$, at v_I^*, is fixed as $\Delta_I^* = 6$m. In Phase 2, the platoon accelerates to another desired velocity v_{II}^*, which is lower than v_0 as they enter an urban area. The desired final spacing between vehicles, denoted as $\Delta_{II}^* = 15$m, is subsequently adjusted based on the safety distance at the new speed v_{II}^*.

Simulations were conduced of two distinct scenarios to assess the performance of the VCG mechanism. In the first scenario (Truthful), all vehicles report their actual expense matrix M_i to the supervisor S. In the second scenario (Misreported), each vehicle V_i alternatively declares $\tilde{M}_i$ to the supervisor S.

Figure 1 depicts the time histories of each vehicle's velocity, position, and inter-vehicle distance relative to the leader in the Truthful scenario. These plots collectively illustrate the platoon's behavior during both the deceleration and acceleration phases. Each vehicle successfully coordinates and precisely attains not only the common optimal speeds but also the desired spacings. Specifically, area $\mathbf{Z}_I$ highlights the correct achievement of the desired spacing Δ_I^* at the conclusion of Phase I, while area $\mathbf{Z}_{II}$ demonstrates the attainment of the desired spacings Δ_{II}^* at the conclusion of Phase II.

Table 2 presents the cost $\mathcal{C}_i$ associated with each vehicle V_i, evaluated as the sum of individual costs incurred during Phases I and II. These costs are computed based on both the actual expense matrix M_i and the declared expense matrix $\tilde{M}_i$, leveraging on the formulas (5) and (7) under the initial conditions specific to each respective phase. The substantial cost discrepancies observed in Table 2 illustrate the efficacy of the mechanism in penalizing misreporting, thereby rendering truthful reporting the optimal strategy for each vehicle, as established in Theorem 2. The increased costs incurred by vehicles that misreport serve to deter strategic manipulation, ensuring the efficient operation of the platoon.

Note that in the simulation setup—composed of two distinct phases—the evaluation performed by the supervisor for both payments and control in each phase was completed in approximately 0.11 sec on a machine with an i7 processor and 32 GB of RAM. Hence, the approach can be applied within a multistage framework by the supervisor in a one-shot manner, using the analytical evaluation (7) at the initial time of each stage (or phase), treating them separately. In the considered scenario, the maximum relative error between the analytical and numerical evaluation of (7) in each stage is less than 0.1%. This enables greater modularity and adaptability in practical deployment, making the proposed LQ-based VCG mechanism well-suited for scenarios involving larger platoons.

6. CONCLUSION AND FUTURE WORK

This paper presents a VCG-based LQ control framework for vehicle platooning that ensures truthful cost reporting and optimizes performance. Key contributions include: 1. LQ control with CARE-based stability, 2. a VCG mechanism for incentive compatibility, and 3. a scalable design for heterogeneous fleets. Simulations demonstrate successful platoon formation, improved comfort and mitigation of strategic misreporting. Future work will extend the framework to larger platoons and incorporate imperfect information arising from communication delays and sensing errors, thereby enhancing robustness and practical applicability.

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Table 1. Platoon Simulation Parameters

Parameter	Symbol	Value(s)	Units
Number of vehicles	n	5	
Simulation duration		100	s
Vehicle masses		$[1800, 1500, 2200, 1200, 2000]$	kg
Damping coefficients		$[250, 200, 300, 150, 280]$	Ns/m
Position cost weights	q_i^p	$[10, 10, 10, 10, 10]$	
Velocity cost weights	q_i^v	$[5, 5, 5, 5, 5]$	
Control weights	r_i	$[200, 100, 50, 30, 200]$	
Position coupling weights	γ_i	$[5, 2, 2.5, 3, 3.5] \times 10^5$	
Velocity coupling weights	β_i	$[180, 80, 100, 120, 140]$	
Starting cruise speed	v_0	28	m/s
Desired velocity, Phase I	v_I^*	0.3	m/s
Desired velocity, Phase II	v_{II}^*	14	m/s
Desired spacing, Phase I	Δ_I^*	6	m
Desired spacing, Phase II	Δ_{II}^*	15	m

Table 2. Total Vehicle Costs (Truthful vs Misreported)

Vehicle	Truthful Cost	Misreported Cost	Δ Cost
1	1.36×10^{10}	1.47×10^{10}	1.13×10^9
2	5.26×10^9	5.50×10^9	2.72×10^8
3	6.87×10^9	7.37×10^9	5.06×10^8
4	6.23×10^9	6.62×10^9	3.94×10^8
5	1.73×10^{10}	1.85×10^{10}	1.23×10^9

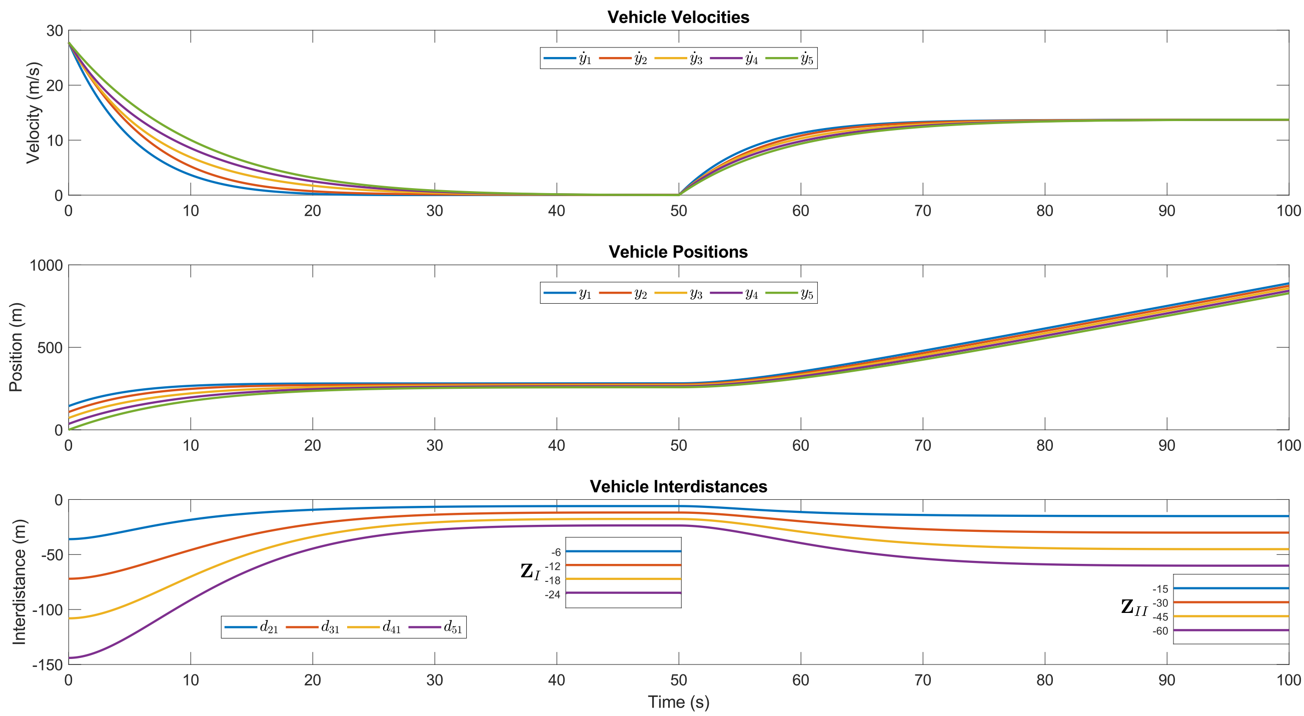

Fig. 1. Time histories of vehicle dynamics in the Truthful scenario, illustrating the platoon's coordination during Phase I and Phase II.